

Typing errors in the second expanded Edition of Mumford's Red Book of Varieties and Schemes

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Abstract

In this article i try to correct some typing errors in the second expanded edition of David Mumford's red book of varieties and schemes, Springer 1999.

I've bought my red book in 2003 and started working with it. I corrected the errors primary just in my book. In order to make them accessible for other mathematicans i \LaTeX ed them now.

I didn't count the lines to make the errors precise. I just reference to the top, middle and bottom of a page to locate an error.

I didn't work out the III§3 ff in detail, so no errors corrected in these Paragraphs.

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Feel free to mail me corrections and further errors you find in the book and in my list. Thanks in advance. Johann Schuster, info@johann-schuster.de

- p. 2, bottom: ...*new terms* $a \cdot y_1^{b_1+\dots}$ instead of ...*new terms* $a \cdot Y_1^{b_1+\dots}$
- p. 2, bottom: *Therefore*, $f(y_1, z_2 + y_1^{r_2}, \dots, z_m + y_1^{r_m}) = b \cdot y_1^N$ instead of *Therefore*, $f(Y_1, Z_2 + Y_1^{r_2}, \dots, Z_m + Y_1^{r_m}) = b \cdot Y_1^N$
- p. 3, middle: *ideals* $P' \subset R$ instead of *ideals* $P' \subset S$
- p. 5, bottom: I think one should write

$$B = A \cdot k[X_1, \dots, X_n, X_{n+1}] + (1 - g \cdot X_{n+1}) \cdot k[X_1, \dots, X_n, X_{n+1}]$$

instead of

$$B = A \cdot k[X_1, \dots, X_n, X_{n+1}] + (1 - g \cdot X_{n+1})$$

See the proof of Theorem 1.9 (Strong Hilbert Nullstellensatz) in [1]

- p. 6, middle: *assumption on* instead of *assumptionon*

- p. 6, middle:

$$\sum_{i=1}^m h_i(X_1, \dots, X_{n+1}) \cdot f_i(X_1, \dots, X_n)$$

instead of

$$\sum_{i=1}^m h_i(X_1, \dots, X_{m+1}) \cdot f_i(X_1, \dots, X_n)$$

- p. 7, top: $x \notin V(A) \cup V(B)$ instead of $c \notin V(A) \cup V(B)$
- p. 9, bottom: \S instead of $\S\S$
- p. 12, middle: *twisted cubic in $\mathbb{P}_3(k)$* instead of *twisted cubic $\mathbb{P}_3(k)$*
- p. 12, middle: $xw = yz$ instead of $zw = yz$
- p. 13, middle: Here one could write $g \circ \alpha$ instead of $g \cdot \alpha$ twice
- p. 13, middle: *Let f_i* instead of *Let f_1*
- p. 16, top: *not equal to P_0 or P_1* instead of *not equal to P_1 or P_2*
- p. 20, middle: *suppose $F \in \underline{\mathcal{O}}_X(X_f)$* instead of *suppose $f \in \underline{\mathcal{O}}_X(X_f)$ and imply $F \in R_f$* instead of *imply $f \in R_f$*
- p. 23, bottom: *induced sheaf $\underline{\mathcal{O}}_Y$* instead of *induced sheaf $\underline{\mathcal{O}}_X$*
- p. 24, bottom: *is the coordinate* instead of *is the coordinater*
- p. 26, top: *nonempty open set in W_1* instead of *nonempty open set in W*
- p. 29, middle: *take $F(\tilde{x})/G(\tilde{x})$* instead of *take $F(\tilde{x})/(G(\tilde{x}))$*
- p. 29, middle: $\varphi : X \cap \mathbb{P}_n(k)_{X_0}$ instead of $\varphi : X \cap \mathbb{P}_n(k)_{X_n}$
- p. 29, bottom: I wrote $\underline{\mathcal{O}}_{\varphi(x)}$ instead of $\underline{\mathcal{O}}_{\varphi x}$ twice
- p. 35, bottom: At the end of the proof, right before *Remarks*, a \square is missing
- p. 36, middle: *I takes (S_i, T_j)* instead of *I takes (s, t)* and $R_{ij} = S_i T_j$, $R_{i0} = S_i$, $R_{0j} = T_j$ instead of $R_{ij} = s_i t_j$, $R_{i0} = s_i$, $R_{0j} = t_j$
- p. 38, bottom: *let Γ_f be this image* instead of *let Γ_g be this image*
- p. 38, bottom: In the diagram: $p_{1|\Gamma_f}$ instead of $P_{1|\Gamma_f}$
- p. 39, top: *neighbourhood of z* instead of *neighbourhood of Z*
- p. 39, top: I wrote *all $x, y \in X \subseteq \mathbb{P}_n(k)$* instead of *all $x, y \in \mathbb{P}_n(k)$*

- p. 41, bottom: in Proposition 2 $f : X \rightarrow Y$ instead of $f : X \rightarrow Z$
- p. 47, top: End of Corollary (*resp.* $X \cap V((f_1, \dots, f_r)) = \emptyset$) instead of (*resp.* $X \cap V((f_1, \dots, f_r)) = \emptyset$)
- p. 47, middle: *Assume* $W = f(\mathbb{P}_n)$ instead of *Assume* $W = f(\mathbb{P}_m)$
- p. 47, bottom: *Also, by Cor. 3** instead of *Also, by Cor. 2**
- p. 50, bottom: *for all* $y \in U$ instead of *for all* $y \in Y$
- p. 51, middle: W_{i_1}, \dots, W_{i_s} instead of W_{i_l}, \dots, W_{i_s}
- p. 52, top: W_{i_1}, \dots, W_{i_s} instead of W_{i_l}, \dots, W_{i_s}
- p. 52, bottom: $g' = f^*g$ instead of $g' = f * g$
- p. 53, middle: $k[T] \supseteq k[T^2, T^3]$ (or depending on the definition just $k[T] \supset k[T^2, T^3]$) instead of $k[T] \not\supseteq k[T^2, T^3]$. To be consistent, lateron i'll correct this always like the first case.
- p. 55, middle: $f(X_0, \dots, X_n)$ instead of $f(X_0, \dots, X_m)$
- p. 62, top: $\bar{V} \cap (S \times V)$ instead of $\bar{V} \subset (S \times V)$
- p. 66, middle: $P \subsetneq R$ instead of $P \not\subset R$
- p. 67, bottom: $V(A) \supseteq V(B)$. *Similarly* $V(A) \supseteq V(C)$ instead of $V(A) \not\supseteq V(B)$. *Similarly* $V(A) \not\supseteq V(C)$
- p. 69, middle: *natural map* $R_f \rightarrow R_P$ instead of *natural map* $R_f \rightarrow R_p$
- p. 69, bottom: *goes to 0 in* R_P instead of *goes to 0 in* R_p and R_P instead of R_p in the diagram
- p. 69, bottom: *follows* instead of *folllows*
- p. 70, top: *Image*(g) instead of *image* (g)
- p. 70, middle: $N = n + M$ instead of $N = m + M$
- p. 70, middle: $b'_i f_j^N - b'_j \cdot f_i^N = 0$ *in* R instead of $b'_i f_j^N - b'_j \cdot f_i^N = 0$ *in* R
- p. 70, middle:

$$X = \bigcup_{i=1}^k X_{f_i} = \bigcup_{i=1}^k X_{f_i^N}$$

instead of

$$X = \bigcup_{i=1}^k X_{f_i} = \bigcup_{i=1}^k X_{f_i} N$$

- p. 70, middle: *Therefore* $1 \in (f_1^N, \dots, f_k^N)$ instead of *Therefore* $1 \in (f_1^N, \dots, f_K^N)$
- p. 79, top: $\underline{o}_y/m_y \xrightarrow{\overline{f_x^*}} \underline{o}_x/m_x$ instead of $\underline{o}_y/m_y \xrightarrow{\overline{f_x^*}} \underline{o}_x/m_x$
- p. 80, middle: $f([P]) = [A^{-1}(P)]$ instead of $f([P]) = [a^{-1}(P)]$
- p. 81, middle: *But by Prop. 1 of §1* instead of *But by Prop. 1 of §4*
- p. 82, top: $P \subsetneq M$ instead of $P \not\subseteq M$
- p. 85, middle: $p_1^{-1}(U_{k,i} \cap U_{k',i'}) \cap p_2^{-1}(V_{k,j} \cap V_{k',j'}) \subset \Phi_{k,i,j}$ instead of $p_1^{-1}(u_{k,i} \cap U_{k',i'}) \cap p_2^{-1}(V_{k,j} \cap V_{k',j'}) \subset \Phi_{k,i,j}$
- p. 90, middle: *Then* $\{y\}$ instead of *Then* (y)
- p. 90, middle: $U \rightarrow U \cap X$ instead of $U \rightarrow U \cap V$
- p. 92, middle: $\text{Spec}(A_i \otimes_k B_j)$ instead of $\text{Spec}(A_i \otimes_j B_j)$
- p. 92, bottom: $y_1 = 1/x_1$ instead of $y_1 = 1/x_2$
- p. 93, top:

$$k \left(\frac{x_1}{x_0}, \frac{x_2}{x_0} \right) \simeq k \left(\frac{y_1}{y_0}, \frac{y_2}{y_0} \right)$$

instead of

$$\left(\frac{x_1}{x_0}, \frac{x_2}{x_0} \right) \simeq k \left(\frac{y_1}{y_0}, \frac{y_2}{y_0} \right)$$

and

$$x_1/x_0 \mapsto y_0/y_1$$

instead of

$$x_1/x_0 \mapsto y_1/y_1$$

- p. 95, top: $k_0[X_1, \dots, X_n]/(f_1, \dots, f_m)$ instead of $k_0[X_1, \dots, X_n](f_1, \dots, f_m)$
- p. 97, top: either *(a) and (b)* or *a) and b)*
- p. 98, bottom: In the upper line of the diagram there should be

$$\prod_i (\Gamma(U_i, \underline{o}_{X_0}) \otimes_{k_0} k)$$

instead of

$$\prod_i (\Gamma((U_i, \underline{o}_X) \otimes_{k_0} k)$$

and in the lower line $\Gamma(p^{-1}(U), \underline{o}_X)$ instead of $\Gamma(p^{-1}(U)\underline{o}_X)$

- p. 100, top: *all* $x \in p^{-1}(y)$ instead of *all* $x \in p^{-1}(u)$

- p. 100, middle: x 's instead of X 's
- p. 102, middle: *Since* $\mathbb{R}[X, Y]/(X^2 + Y^2)$ instead of *Since* $\mathbb{R}[X, Y]/(X^2 - Y^2)$
- p. 103, bottom: *we have* $\mathcal{L}_x = \mathcal{G}_x/\varphi_x(\mathcal{F}_x)$ instead of *we have* $\mathcal{L}_x = \mathcal{G}_x/\varphi_*(\mathcal{F}_x)$
- p. 104, bottom: *A closed subscheme of* X instead of *A closed prescheme of* X
- p. 106, top:

$$\ker \left\{ \Gamma \left((\text{Spec } R)_f, \mathcal{O}_{\text{Spec}(R)} \right) \rightarrow \Gamma \left((\text{Spec } R)_f, \mathcal{O}_{\text{Spec}(R/A)} \right) \right\}$$

instead of

$$\ker \left\{ \Gamma \left((\text{Spec } R)_f, \mathcal{O}_{\text{Spec}(R)} \right) \rightarrow \left((\text{Spec } R)_f, \mathcal{O}_{\text{Spec}(R/A)} \right) \right\}$$

- p. 106, middle: *hence* $V(B) = V((0))$ instead of *hence* $V(B) = B((0))$
- p. 108, top: $\mathbb{A}^1 = \text{Spec}(k[t])$ instead of $A^1 = \text{Spec}(k[t])$
- p. 108, top: $\mathcal{O}_{a_i, Y} = \mathcal{O}_{a_i, \mathbb{A}^1}/m_i^{r_i}$ instead of $\mathcal{O}_{a_i, y} = \mathcal{O}_{a_i, \mathbb{A}^1}/m_i^{r_i}$
- p. 108, bottom: *Let* $Y = \text{Spec}(k[x, y]/A)$ instead of *Let* $Y = \text{Spec}([x, y]/A)$
- p. 110, top and middle: Z_j and I_j instead of Z_1 and I_1 . Found eight times I_1 and five times Z_1
- p. 112, middle: *Then for any manifold* X instead of *Then for any manifold* Z
- p. 112, middle: $\text{hom}_C(z, G) \cong G$ *as a point set* instead of $\text{hom}_C(Z, G) \cong H$ *as a point set*
- p. 113, top: *Examples* Q and S instead of *Examples* 1 and 4
- p. 113, middle: *to* $\text{hom}_C(z', X)$ instead of *to* $\text{hom}_X(z', X)$
- p. 114, middle: $X = \text{Spec}(\mathbb{Z}[X_1, \dots, X_n]/(f_1, \dots, f_m))$ instead of $X = \text{Spec}(\mathbb{Z}[X_1, \dots, X_n]/(f_1, \dots, f_M))$
- p. 115, bottom: *i.e.,* $F(q)$ *injective, and* $\text{Im } F(q) =$ instead of *i.e.,* $F(g)$ *injective, and* $\text{Im } F(g) =$
- p. 116, top: *there is a morphism* g : instead of *there is a morphism* q :
- p. 116, middle: *with image* x instead of *with image* X

- p. 116, middle:

$$\left\{ \begin{array}{l} \text{set of } k\text{-valued} \\ \text{points of } X \end{array} \right\}$$

instead of

$$\left\{ \begin{array}{l} \text{set of } k\text{-valued} \\ \text{points of } x \end{array} \right\}$$

- p. 118, bottom: $Z_2 = \{x \in K \mid \pi(f(x)) \equiv \pi(g(x))\}$ instead of $Z_2 = \{x \in K \mid \pi(f(x)) \equiv \pi((x))\}$
- p. 124, middle: *If* $X \subset \mathbb{A}_k^2$ instead of *If* $x \in \mathbb{A}_k^2$
- p. 138, middle: *open set* X_f instead of *open set* X_d
- p. 138, middle: *natural map* $M_g \rightarrow M_f$ instead of *natural map* $M_h \rightarrow M_f$
- p. 139, middle:

$$\Gamma(U, \mathcal{F}) \otimes_{\Gamma(U, \mathcal{O}_X)} \Gamma(V, \mathcal{O}_X) \rightarrow \Gamma(V, \mathcal{F})$$

instead of

$$\Gamma(U, \mathcal{F}) \otimes_{\Gamma(U, \mathcal{O}_X)} \Gamma(X, \mathcal{O}_X) \rightarrow \Gamma(X, \mathcal{F})$$

- p. 140, middle: $\mathcal{F} |_{U_f} \cong \widetilde{M}_f$ instead of $\mathcal{F} |_{U_f} \cong M_f$
- p. 140, bottom: $\mathcal{F} |_{U_{g_i}} \cong \widetilde{N}_i$ instead of $\mathcal{F} |_{U_{g_i}} \cong N_i$
- p. 140, bottom: *sheaves* \mathcal{F}_i^* instead of *sheaves* \mathcal{F}_i^*
- p. 143, top: $\tau(\sum \alpha_i d\beta_i)$ instead of $\tau(\sum \beta \alpha_i d\beta_i)$
- p. 143, middle: *define a ring* $E =$ instead of *define a ring* $R =$
- p. 149, middle: In the diagram the line p_2 should be an arrow to U_i
- p. 152, middle: $\mathcal{F}_x \otimes_{\mathcal{O}_x} \mathbb{k}(x) = (0)$ instead of $\mathcal{F}_x \otimes_{\mathcal{O}_X} \mathbb{k}(x) = (0)$
- p. 152, middle: *generate* $\mathcal{F}_x \otimes_{\mathcal{O}_x} \mathbb{k}(x)$ instead of *generate* $\mathcal{F}_x \otimes_{\mathcal{O}_x} \mathbb{k}(x)$

References

- [1] J.S. Milne: *Algebraic Geometry*, <http://www.jmilne.org/math/>, Version 4.00, October 2003